A GENERAL FUNCTIONAL ANALYSIS TO DISPERSIVE STRUCTURES

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ABSTRACT

A new functional for 3-Dimensional waveguiding structures with dielectric and conductor losses is rigorously derived. Numerical examples are presented for anisotropic dielectric image waveguide, PTEE bilateral fin line, and two coupled asymmetrical dual lossy transmission lines with finite conductivity and finite thickness. Agreement with previous publications wherever available are observed.

INTRODUCTION

Based on the vector element concept, the full vector functional has been derived and successfully applied to the lossless guided wave structures [1]. Recently, the three field component functional has further been developed for the study of the lossy guided wave structure [2]. In these studies, the adjoint field in the functional formulation is either the working variable itself or the complex conjugate of the working variable.

In this presentation, the unified three field component functional is formulated in a natural way from the 3D configuration. The new functional can be applied to the general 2.5 dimensional guided wave structures, including lossy or lossless, isotropic or anisotropic cases, and even with the inhomogeneous boundary conditions.

BASIC THEORY

The 3D functional can be expressed as

$$
\begin{aligned}
I &= \int_v [\frac{1}{\mu_r}(\nabla \times \vec{E}) \cdot (\nabla \times \vec{E}^\dagger) - k_0^2 \vec{E}^\dagger \cdot \vec{\epsilon}_r \cdot \vec{E}] dv \\
&+ j\omega\mu_0 \int_v [\vec{J}^e \cdot \vec{E}^\dagger + \vec{J}^{e\dagger} \cdot \vec{E}] dv \\
&+ \int_{s2} \gamma_p (\hat{n} \times \vec{E}^\dagger) \cdot (\hat{n} \times \vec{E}) ds \\
&+ \int_{s2} (\vec{U} \cdot \vec{E}^\dagger + \vec{U}^\dagger \cdot \vec{E}) ds
\end{aligned} \qquad (1)
$$

subject to the following boundary conditions:

$$
\begin{cases} \hat{n} \times \vec{E} = \vec{P} & \text{on } S_1 \\ \frac{1}{\mu_r} \hat{n} \times \nabla \times \vec{E} + \gamma_p \hat{n} \times \hat{n} \times \vec{E} = \vec{U} & \text{on } S_2 \end{cases} \qquad (2)
$$

where $S = S_1 \cup S_2$, γ_p is a frequency dependent parameter and $\vec{U}$ could be a generalized known function, according to the local potential concept [5]. When analyzing the propagation modes for the guided wave structures, where uniformity is assumed in the wave propagation direction, the surface integration on O_1 and O_2, as shown in Fig. (1) should vanish since the contributions from these terms are z coordinate dependent. If the adjoint field is chosen

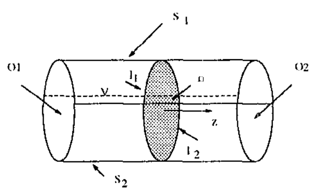

Fig. 1. 3D structure

to be the field which propagates in the opposite direction, the contribution from the surface integration on O_1 would cancel that on O_2. Based on [3], the original system and the special adjoint system can be expressed as

$$
\begin{aligned}
\vec{E} &= (\vec{E}_t + \hat{z}E_z)e^{-\gamma z} \\
\vec{E}^\dagger &= (\vec{E}_t - \hat{z}E_z)e^{\gamma z}
\end{aligned} \qquad (3)
$$

By introducing new variables [1]

$$
E_z = \gamma e_z \qquad \vec{E}_t = \vec{e}_t
$$

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we end with

$$
\begin{aligned}
I_p(\vec{e}) &= \int [\frac{1}{\mu_r}(\nabla_t \times \vec{e}_t)^2 - k_0^2 \vec{e}_t \cdot \vec{\vec{\epsilon}}_{tt} \cdot \vec{e}_t] d\Omega \\
&- \gamma^2 \int \frac{1}{\mu_r} [(\vec{e}_t + \nabla_t e_z)^2 - \epsilon_{rz} k_0^2 e_z^2] d\Omega \\
&+ \int_{l_2} \gamma_p [(\vec{n} \times \vec{e}_t) \cdot (\vec{n} \times \vec{e}_t) - \gamma^2 e_z^2] dl
\end{aligned}
\tag{4}
$$

where I_p is the functional per unit length, and $l = l_1 \cup l_2$, is the contour of the cross section Ω. Source free is also assumed in the expression without losing generality. Equation (4) is the desired form in describing the eigenvalues and eigenvectors of the general $2\frac{1}{2}$ dimensional dispersive and anisotropic waveguiding structures. This new functional therefore can handle problems with conductors of finite conductivity and finite cross sections in a lossy anisotropic dielectric environment under inhomogeneous boundary conditions, provided $\vec{U} = 0$ on the side enclosure. When the ground plane consists of imperfect conductors, the surface impedance boundary condition is applicable. In this case,

$$
\gamma_p = jk_0 \sqrt{\frac{\epsilon_{rc} - j\frac{\sigma}{\omega\epsilon_0}}{\mu_{rc}}} \tag{5}
$$

where ϵ_{rc} and σ are respectively the relative permittivity and conductivity of the thick imperfect conductor. This relation is also valid at the interface between the thick imperfect conductor and the anisotropic dielectric material.

CIRCUIT PARAMETERS

Besides the complex propagation constant, which can be evaluated using (4), the complex characteristic impedance is another useful parameter. In the engineering applications, equivalent circuit parameters are preferred [4]. Once the eigenvalues and the eigenvectors are obtained from the edge element procedure, the circuit parameters can be extracted based on field superposition principle and complex average power equivalence assumption. We have,

$$
[Z_c] = \begin{cases} [V]\,[I]^{-1} \\ [I^{*t}]^{-1}\,[P^m]^t\,[I]^{-1} \\ [V]\,[P^{m*}]^{-1}\,[V^{*t}] \end{cases} \tag{6}
$$

$$
\begin{aligned}
[Z_{cir}] &= [R] + j\omega\,[L] \\
&= \begin{cases} [V]\,[\Gamma]\,[I]^{-1} \\ [I^{*t}]^{-1}\,[P^m]^t\,[\Gamma]\,[I]^{-1} \\ [V]\,[\Gamma]\,[P^{m*}]^{-1}\,[V^{*t}] \end{cases}
\end{aligned}
\tag{7}
$$

and

$$
\begin{aligned}
[Y_{cir}] &= [G] + j\omega\,[C] \\
&= \begin{cases} [I]\,[\Gamma]\,[V]^{-1} \\ [V^{*t}]^{-1}\,[P^{m*}]\,[\Gamma]\,[V]^{-1} \\ [I]\,[\Gamma]\,[P^m]^{-t}\,[I^{*t}] \end{cases}
\end{aligned}
\tag{8}
$$

where, $[I]$ has the entries of

$$
I_{ik} = \oint_i \vec{H}^k \cdot d\vec{l} \tag{9}
$$

The voltage matrix is defined as

$$
V_{ik} = \int_i \vec{E}^k \cdot d\vec{l} \tag{10}
$$

and the power matrix is found to be

$$
\{P^m\}_{i,j} = \iint \vec{E}_i^m \times \vec{H}_j^{m*} \cdot d\vec{s} \tag{11}
$$

At high frequencies, the circuit parameters extracted from $P - I$ definition may be different from those of $P - V$ definition.

NUMERICAL RESULTS

The edge element method and the subspace iteration method are used to find the saddle points of the functional. The desired propagation constants and the associated electromagnetic fields distribution are obtained consequently. Numerical examples are provided to demonstrate the effectiveness, versatility, and generality of the new formulas, and to verify that the spurious modes have been suppressed.

Example 1 Lossy Image Waveguide

A lossy anisotropic dielectric loaded image waveguide, as shown in Fig. (2), is presented as our first example. This structure has been studied by other researchers using different methods. In the computation, we select the real part of ϵ_{yy} as a parameter to study the effect on the propagation modes. The numerical results are the same as with the others, but the basic formulation is much simpler and the effect of the lossy boundary has been included. Fig. (3) is the attenuation curve, and Fig. (4) is the propagation constant curve. Each curve shown in these figures takes only a few seconds of CPU time on the DEC AXP 3000 machine.

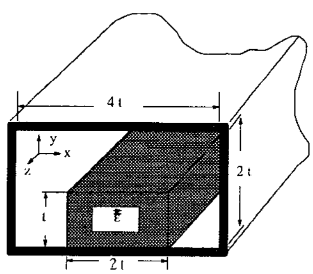

Fig. 2. lossy image waveguide

Example 2 Bilateral Finline

As shown in Fig. (5), a bilateral finline constructed on PFEE material is studied. Both the finline and the waveguide are made of copper with the conductivity $\sigma = 5.72 * 10^7\ S/m$. Fig. (6) provides the attenuation constants due to ohmic losses. Fig. (7) shows the propagation constants versus frequency.

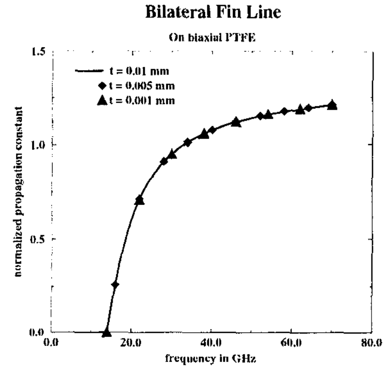

Fig. 3. propagation for a lossy image waveguide

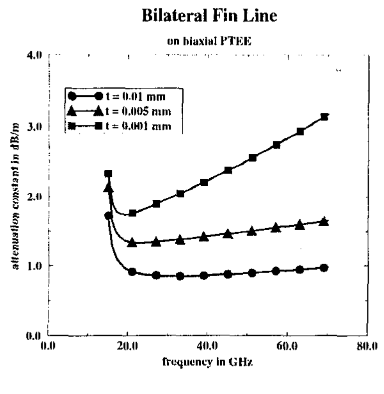

Fig. 4. attenuation for a lossy image waveguide

Example 3 Coupled Asymmetrical Lossy Microstrips

In principle, transmission lines with both ohmic and dielectric losses may be modeled by the full-wave method using integral equation approaches. However, it may be difficult to find the Green's functions for complicated geometries. Even if the Green's function is found, it may be uneasy to obtain the complex propagation constants of the fundamental modes, because the corresponding eigen equations are transcendental equations with the unknown eigenvalues being inexplicit parameters. In contrast, the edge element method has geometric versatility. We present a coupled asymmetrical lossy microstrip line system, shown in Fig. (8), as the third example to illustrate the application of the new functional. In this example, all the circuit parameters, such as, L, R, C, and G, as well as the characteristic impedance matrix, are calculated according to our newly derived formulation.

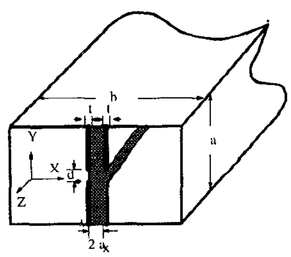

Fig. 5. bilateral finline configuration

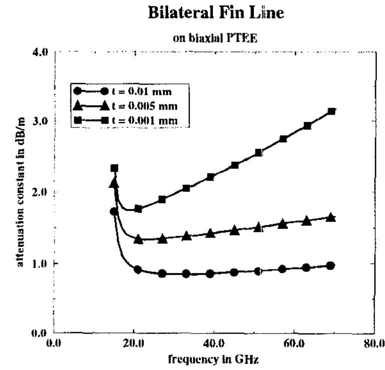

Fig. 6. attenuation constant of a bilateral finline

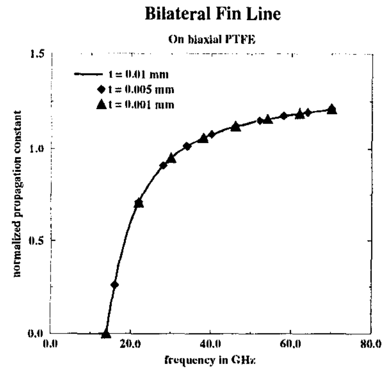

Fig. 7. propagation constant of a bilateral finline

CONCLUSION

In this presentation, a new functional for 3 dimensional structures is derived and applied to a variety of 2.5D guided wave devices. Ohmic and dielectric losses are treated systematically and consistently under the full wave regime. An extended boundary condition of the third kind is proposed and employed for the open structures to confine the computational region with success. The subspace iteration method is used to handle large scale generalized complex eigenvalue problems. Numerical examples of waveguides and transmission lines for digital and millimeter wave applications are presented.

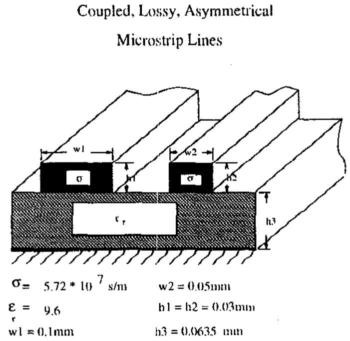

Fig. 8. dual microstrip lines

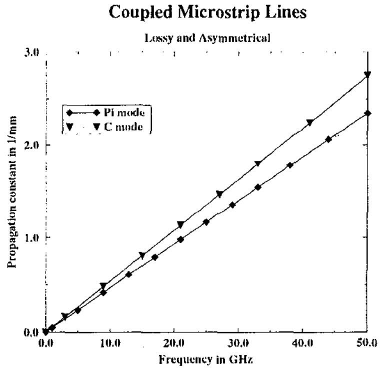

Fig. 9. propagation modes

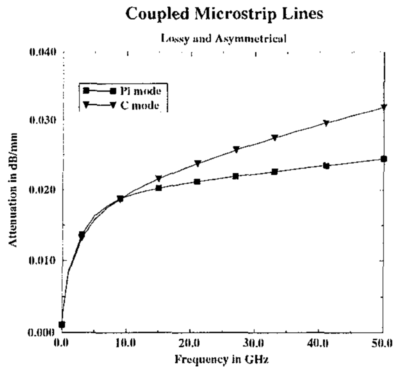

Fig. 10. attenuation modes

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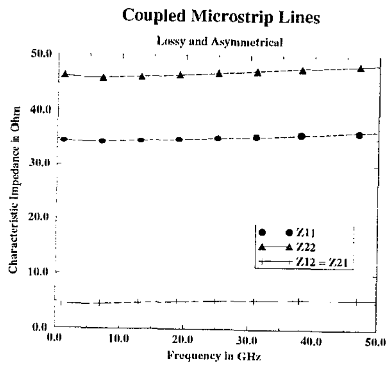

Fig. 11. characteristic impedances

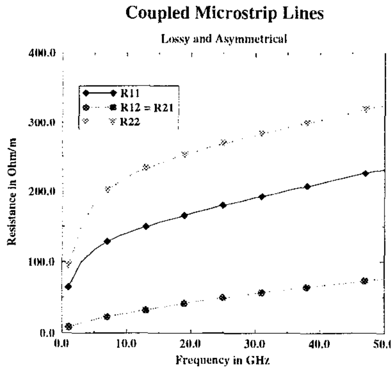

Fig. 12. line resistance

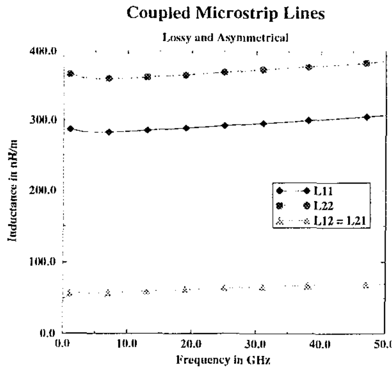

Fig. 13. line inductance

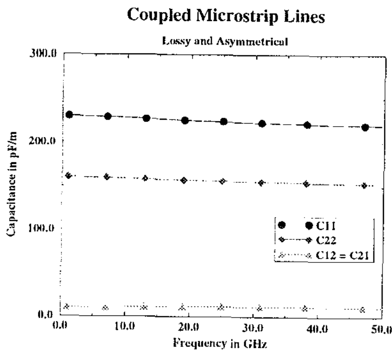

Fig. 14. line capacitance

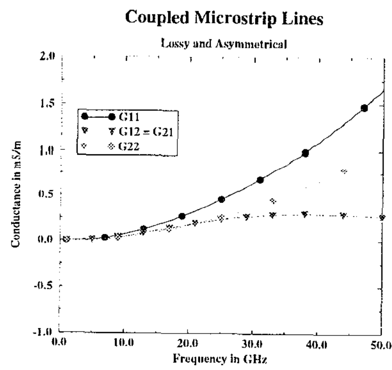

Fig. 15. line conductance